Geometry and digital cultural heritage as unique linking for developing students’ knowledge, skills and attitudes

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ABSTRACT

This paper presents the fruitful connection between geometry and cultural heritage as an innovative teaching and learning approach and examines the way that students learn geometrical concepts and in which way they construct and transform geometrical concepts and students’ engagement to get familiar with cultural heritage. This pilot research based on implementation of learning scenario by Europeana DSI-4 project and results provided by qualitative and quantitative evaluation could motivate the educational community for implementation of interdisciplinary learning scenarios with geometry for developing students 21st century skills through cultural heritage.

SINTESI

Lo studio presenta la fruttuosa connessione tra lo studio della geometria e del patrimonio culturale come approccio didattico innovativo, ed esamina il modo in cui l'apprendimento di concetti geometrici promuova la familiarizzazione degli studenti con il patrimonio culturale. Lo studio pilota è basato sull’implementazione di scenari di apprendimento da parte del progetto Europeana DSI-4. I risultati tratti da una valutazione qualitativa e quantitativa possono motivare le comunità educative ad implementare scenari di apprendimento interdisciplinari includendo la geometria, per lo sviluppo delle skills per il XXI secolo, attraverso il patrimonio culturale.

KEYWORDS: geometry, cultural heritage, Europeana, interdisciplinarity

PAROLE CHIAVE: geometria, patrimonio culturale, Europeana, interdisciplinarietà
Introduction: statement of educational challenges

Cultural heritage is an “invaluable treasure”, through the centuries that connects the present with the past and conveys ethical values, defining the identity of each country. Students as members of the modern world, it is very important to have knowledge of the historical perspective of the heritage that provides information about what you can expect in the future, so that it may serve as a guide in making future decisions (Ott & Pozzi, 2011). The preservation of cultural heritage can also contribute to education for sustainable development, strengthening the role of education for environmental protection (Penna, 2018; Gilmour, 2006).

However, according to Gesche-Koning (2018) educational systems do not link curriculum with cultural heritage. Specifically, there is 1) Lack of a structural timetable for cultural heritage education in current school educational systems (Culture et Démocratie, 2009; 2011); 2) Failure to disseminate sufficient transferable examples in the past and present (Collard & Witte, 2015); 3) Insufficient systematic training in cultural heritage education (Cramer, 2003). The integration of cultural heritage in the teaching of the subjects of the curriculum raises the interest of the research community (Gómez-Carrasco et al., 2020; Karadeniz & Çildir, 2017), while at the same time an effort is made to develop initiatives to support teachers1.

On the other hand, there is a strong interest in the educational and research community in enriching learning and teaching strategies where they will improve the understanding of the basic concepts of geometry and expand to help cultivate and develop students’ geometric reasoning (Ma, Lee, Lin & Wu, 2015; Willis, 2007; NCTM, 2000). Although geometry is an integral part of the curriculum, many students fail to develop an in-depth understanding of the basic concepts of geometry (Namkung et al., 2019; Howse & Howse, 2015; Hallat, 2008; Hittleman & Simon, 2002). More specifically, from the research literature it appears that the difficulties of students in learning geometry are related to teaching methods, the lack of understanding of the proof process, the lack of visualization of geometric concepts, the inability to understand and use language and especially the lack of educational material, the enrichment of teaching practices (Mifetu et al., 2019; Fabiyi, 2017; Uduosoro, 2011; Telima, 2011; Argyri, 2012; Aysen, 2012; Argyri, 2010).

Taking under consideration the above educational challenges, this paper focus on provide methodological teaching methods of geometry in order to remove the difficulties in understanding the geometric concepts of students and promote the cultivation of geometric reasoning and skills, through the qualitative and quantitative assessment of the interdisciplinary connection between geometry and cultural heritage. The pilot implementation is based on solving geometrical

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problems with real situations with mediation tools of the cultural heritage (works of art, architectural buildings, etc.).

1. Theoretical framework

1.1. Educational aspects of cultural heritage

Culture can be defined as the beliefs, values, attitudes, customs, social relations, art and literature that determine the identity of each country (Abidi, 1996; Banks, 2008). Cultural heritage includes tangible cultural heritage: movable cultural heritage (paintings, sculptures, coins, and manuscripts), immovable cultural heritage (monuments, archaeological sites, etc.), and intangible cultural heritage, prehistoric heritage: prehistoric ceremonies. The objects of material culture consist of the tangible cultural heritage (Tilley et al., 2006; Carvalho & da Silva, 2014).

Cultural heritage could offer many educational and participatory opportunities to young people. It could promote dialogue between different cultures and generations, offer a sense of common understanding of differences and similarities, and encourage the appreciation of cultural diversity. According to the European Reference Framework (van Lakerveld et al., 2011), cultural heritage education offers great potential in terms of: 1) The increase and maintain motivation; 2) the innovative interdisciplinary approaches; 3) the European cultural dimension; 4) achieving the transversal core competencies of lifelong learning: learning for learning, social and political skills, a sense of initiative and entrepreneurship, and cultural awareness and expression “leading to” personal fulfillment, active citizenship, social cohesion and employability in a knowledge society.

According to the report of Creative Alliances for Europe (Collard & Witte, 2015) cultural heritage in education enforce the activation and constructivation in citizens of Europe, the development of problem-solving capacity and creativity. Referring to the distinction made between arts in the education (learning about the arts), arts for the education (development of skills, knowledge and techniques required for become an artist) and education through the arts (using the arts’ ability to develop one’s potential and to understand the world).

1.2. Teaching and learning geometry in secondary education

The teaching of geometry is associated with the development of geometrical sense – geometrical thinking, i.e. those mental activities through which students organize and process the elements of lived space to transform them into geometric objects and relationships (Jaguthsing, 2007; Jones, 2002). The development of geometrical thinking is not interpreted simply as recognizing forms or learning terms and concepts but as a connection of spatial experience with a model that represents it (Jones & Tzekaki, 2016). The study of space and the development of geometric thinking is closely linked to a process that can be described simply as “thinking through visual images”, as an ability to read, interpret, transform an “optical thinking” (Laborde, 2015).
Geometry is a key tool for connecting curriculum knowledge with the real world. That we could perceive through intuition, we are called to connect it with geometric rules and formulate in mathematical language the imagined spatial environment (Güven & Kosa, 2008). Geometric representations can help students understand other areas of mathematics, but also connect with other areas of the curriculum (science, geography, art, design and technology) (Argyri, 2013; Furner & Marinas, 2011; Alex & Mammen, 2012; Alex & Mammen, 2016; Argyri, 2013; Argyri, 2014; Argyri, 2015a; Argyri, 2015b; Al-ebous, 2016; Argyri, 2018).

The description of the nature of visual representations, the role of intuition and imagery are among the most basic issues that need to be considered in the teaching of geometry in the evolution of geometrical reasoning (Kalogirou et al., 2013; Adnan et al., 2019). Visualization is considered the ability to «represent, transform, create, communicate, document and reflect on visual information» (Hershkowitz, 1990, p. 75) and pays attention to visual intuition. Visualization is an essential energy for the transfer of “real objects” through an abstract process, i.e. the ability to form and negotiate a mental image necessary for problem-solving in mathematics (Putra et al., 2018; Patsiomitou & Koleza, 2008). Visual images refer to the representation of the visual appearance of an object, e.g. its shape, color and size (Van Garderen, 2006) play a key role in promoting critical thinking (Robinson et al., 2008). Many research studies aim to link visualization, students’ construction of geometric concepts and their definitions, but also the ability to justify-prove by utilizing the use of technological tools and dynamic geometry environments (Patsiomitou, 2011; Patsiomitou & Emvalotis, 2010).

In the process of visualization, geometric shapes are a key component. In particular, Fischbein (1993) states that in the geometric shape there are three categories of mental entities: the definition, the image and the conceptual shape.

One of the most important theories in geometry education is the theory of Duval (2005) for the examination of registers (e.g. geometric shapes, language) used in the field of geometry and their processing in geometry tasks (semiotic approach). According to Duval (1995), the cognitive processes that justify why and how geometry should be taught in school are:

- **imaging process**: refers to the use of representations (e.g. numbers, images, diagrams, symbols) to display, explore or verify different geometric states;
- **construction process**: refers to the construction actions of a geometric concept;
- **reasoning process**: refers to the procedures of proof, explanation and reasoning.

Finally, according to Duval (1999), the cognitions of the process of capturing geometric states:

- **perceptual apprehension**: the physical recognition (shape, representation, size, brightness, etc.);
● **sequential apprehension**: the description of the construction of the geometric shape;

● **discursive apprehension**:
  a) the ability to connect configuration with geometric principles;
  b) the ability to provide a good description, explanation, argumentation, discount, use of symbols reasoning according to statements made with perception consciousness;
  c) the ability to describe numbers through geometric language/narrative texts;

● **operative apprehension**: the modification of a given format in various ways to explore other configurations:
  a) the simplistic way: dividing the set into parts of different shapes and combining these parts into another shape or sub-numbers;
  b) place: differentiating position or position orientation.

However, according to research studies, his teaching strategies do not promote the development of cognitive processes for capturing geometric states through spatial-visual skills (Christou et al.; Yazdani, 2007; Erdogan & Durmus, 2009; Ozerum, 2012).

2. **Methodology**

2.1. **Research design**

After utilizing the research literature 1) on the importance of cultural heritage in learning 2) the characteristics of teaching methodologies that promote geometrical thinking 3) and mainly taking into account the advantages, the materials and the facilities provided by the Europeana project to the educational community, we used the framework of activity theory for implementation the interdisciplinary approach with geometry and objects included in the digital cultural collections. Activity not only plays an important role in the learning of mathematics (Grives & Dale, 2004), but also activity and learning are interactive and interdependent (Jonassen, 2002), where students’ actions are mediated by tools (Barab et al., 2004). The main mediation tool of this research study is the digital cultural repository of Europeana.

2.2. **Europeana as educational tool**

Europeana provides access to approximately 57 million digitized items from more than 3,500 European libraries, museums, archives and galleries. This huge database represents a wide range of media, thematic and linguistic diversity, images/text/audio/video/3D content in art, architecture, maps, movies, natural history, fashion and more than 37 languages. Much of this content – over 20 million items – is freely licensed and can be reused in a variety of research and learning projects.

Europeana’s mission is to transform the world with culture, unlock cultural heritage treasures and make them available online so that all people can use them.
for recreational, professional or educational purposes. The Europeana DSI-4 project (http://fcl.eun.org/europeana-dsi4), implemented and supported by the European School Network (www.eun.org), aims to encourage teachers to share their experience, and focus on three key aspects: 1) preserving cultural heritage and promoting a sense of European identity and culture; 2) providing more support to teachers; 3) promoting innovation in education in the digital age. The digital collections of Europeana provide multiple perspectives on historical, scientific and cultural developments across Europe and beyond. Educators or educational organizations could get familiar and experience the pedagogical potential of digital cultural heritage and promote the cultivation of students’ 21st century skills (Pocze et al., 2019).

2.3. Research questions

- Can the interdisciplinary approach of geometry and cultural heritage enhance learning motivation in both fields of knowledge?
- Can the material objects of cultural heritage be didactic tools for the development of geometrical reasoning and thinking, through the recognition of geometric properties and the problem-solving?
- Does the implementation of interdisciplinary approaches provide the development of students’ 21st century skills?

2.4. Research scene

The interdisciplinary methodological approach was implemented by a hybrid model of learning and teaching (Helms, 2014), as from March to May schools in Greece were closed due to the global crisis from the COVID-19 pandemic. The content was adapted respectively to the curriculum of geometry of the two classes of high school and specifically as a repetition of 1) the properties of the quadrilaterals (rectangle, square, rhombus, trapezoid) and the theorems and propositions associated with them (median orthogonal triangle, a straight line joining the midpoints of the sides of a triangle, a right triangle with an acute angle of 300, a median of a trapezoid); 2) as recognition of the axioms of stereometry.

The reason for the interdisciplinary connection between geometry and cultural heritage is the STEM Discovery Campaign titled “Innovative Trends in Education”, organized by Scientix and it had elements of inspiration from the learning scenarios included in the Europeana project.

The participants were about 20 students from the 1st grade and 10 students from the 2nd grade of the Model High School Evangeliki of Smyrna in Athens, Greece.

2.5. Data Collection

The implementation followed the steps above:

- 1st phase-preparation: students were asked to explore the collection of the digital cultural heritage of the Europeana project (in the Greek
version) in order to prepare a presentation based on the curriculum in which they will recognize and justify properties and theorems of stereometry and planimetry. They could work individually or in groups, which means that they were encouraged to use digital media (Skype, Microsoft Teams) to exchange views and co-create the material.

- During the online meeting (duration 2 hours) presented the alternative ways of using the digital collection offered by the Europeana project through particular examples. Finally the groups of the students presented their work, discussed and commented on them.
- After the online meeting, students were asked to write and solve a geometrical problem based on the objects of material culture that they had chosen. The problems shared in a Google Drive folder so that it could be solved by their peers.

2.6. Data Analysis

The evaluation of the interdisciplinary approach based on analysis of the students’ replies of the questionnaires that used before and after the implementation and closed-ended and open-ended questions were included. However, special importance was given to the qualitative analysis of the students’ projects.

Before the online meeting students were asked to complete in the pre-questionnaire: 1) examples of Greece’s cultural heritage; 2) to rate on a scale of 1 to 5 the importance of cultural heritage; 3) examples of cultural heritage links to school curriculum.

After the online meeting and when the two projects have been completed students were asked in the post questionnaire: 1) for self-assessed skills acquired on a graded scale from 1 to 10; 2) to evaluate on a Likert scale the interdisciplinary approach in the field of geometry and cultural heritage (acquisition of knowledge and formation); 3) to comment on what they liked best and what they did not.

3. Results

3.1. Analysis of questionnaires

Pre-questionnaire

Students were not able to refer to fields of cultural heritage and their knowledge was limited in famous architectural monuments (Acropolis, Parthenon). Their replies did not include traditions, manners, customs or songs, as presented in Figure 1 and this meant that they could not mention characteristics of the cultural heritage of Greece.
Although, most of them agree with the statement that the cultural heritage of each country is invaluable (Figure 2).

The gaps of teaching and learning about cultural heritage in the Greek curriculum are verified by students’ replies in the question to refer for the links with school subjects. Grouping students replies that Figure 3 provides the categories of connection cultural heritage in Greek curriculum.
Examples of students’ replies:
Student 1: «Certainly the history lesson is directly related to the cultural heritage, as the events take place in those places».
Student 2: «Through history we learn about the past of our country and through literature we learn the manners and customs of various places in Greece».

Post-questionnaire

It is very important for the students themselves to evaluate the development skills through the interdisciplinarity learning process and the results of average rating presented in Figure 4. On the one hand the intercultural understanding and the information analysis have the highest level of self-evaluation and on the other hand self-knowledge and the ability to make a reasoned decision have the lowest level.

![Weighted Average](image)

**Figure 4 - Average rates of students’ skills development**

Additionally, students’ replies in Likert scale about their views and attitudes for the interdisciplinarity learning process and the gained knowledge highlight the success of this methodological learning approach. In particular:

- students believe that they learned geometry from real objects (Figure 5);

![Strongly agree, Agree, Neither agree nor disagree, Disagree, Strongly disagree](image)

**Figure 5 - Learning geometry by real objects**
• students gained knowledge for the cultural heritage through geometry (Figure 6).

**FIGURE 6 - LEARNING CULTURAL HERITAGE THROUGH GEOMETRY**

The methodological approach of implementation was extremely interesting for geometry and cultural heritage and students commented on their satisfaction and positive attitudes (Figure 7 and Figure 8).

**FIGURE 7 - ASSESSMENT OF THE APPROACH TO GEOMETRY FROM CULTURAL HERITAGE**

**FIGURE 8 - ASSESSMENT OF THE APPROACH TO CULTURAL HERITAGE FROM GEOMETRY**
Examples of students’ comments:

Student 1: «A lesson that could be introduced in the school curriculum as students are informed about the cultural heritage of their place and at the same time learn geometry».

Student 2: «It was quite interesting because it showed us that two lessons and something out of school curriculum can be combined».

Student 3: «It was very interesting and innovative, as it promoted an alternative approach to cultural heritage».

Analysis of students’ problems on the objects of cultural heritage

The problems above have been select of characteristic examples of the qualitative analysis of students’ projects in which students:

1. investigate, analyse and justify the properties of the quadrilaterals through the critical review of visual representations / art objects;
2. utilize the experience and pre-existing knowledge for the formulation and solution of geometric problems on the objects of art, based on the theorems and propositions of the properties of the quadrilaterals.
### 1st Problem

In the art object of the Figure 9, students mentioned:

- two right triangles and tables are formed between the scaffolding;
- the beam AB joins the middle of the 2 sides of the red triangle and is therefore parallel to the third side and equal to half of it;
- the light blue straight line segment (beam extension) is the median drawn from the right angle of the triangle and is equal to half of the hypotenuse;
- the distance of the centre of gravity C, D from the vertices of the triangles is 2/3 of the corresponding through.

#### FIGURE 9

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1) Consider a triangle KHM and the median of HZ (figure 10). If HZ = KM / 2:

a) Prove that the angle H is right.

b) Formulate the relevant theorem.

c) State its inverse and prove it.

#### FIGURE 10

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2nd problem

The shape is a rectangular parallelepiped, of which each side consists of 3 equal rectangles. The first rectangle contains squares with side 2 cm. While the second and third rectangles contain circles with a radius of 1 cm. The length of each side of the rectangular parallelepiped is equal to 60 cm. If squares and circles are placed every 1 cm, find how many squares and how many circles there are in total in the rectangle? (Assume that squares and circles start and end at the edges of the rectangle).

3rd problem

1) In the square ABGD it is valid that: The diagonals AG and BD are equal, intersect perpendicular to E, bisect and bisect the corners of the square.

2) In the right triangle BZI it holds that: I and K in BZ and BH respectively. So IK is parallel to ZH and equal to half of it

4th problem

1) If the side of the square is equal to 1 cm find the area of the black area;
2) If the edge of the cube is also 1 cm find the volume of the orange area.

Based on results it is highlighting the practice of writing and solving geometric problems by the students themselves (as the only didactic approach to innovation), which tends to fill the gap in the research community to highlight the use of mathematical language in conjunction with the understanding of basic conceptual structures in different contexts, away from the traditional way of solving textbook exercises with the construction or not of geometric shapes.

Finally, we could mention the development of students’ knowledge in a holistic model as:

- **disciplinary knowledge**: basic concepts and properties that characterize the types of quadrilaterals and the relationships between them;
- **interdisciplinary knowledge**: ability to transfer and recognize knowledge and problems in the objects of digital cultural heritage through the different disciplinary lenses of geometry and art;
- **procedural knowledge**: understanding how problems and exercises for quadrilateral properties are solved (sequence of steps or actions).

**Conclusion**

The interdisciplinary approach of geometry with digital cultural heritage based on Europeana collection in the framework of activity theory via distance learning provides the active and constructive participation of students that require effective and flexible strategies, where they set their own learning goals and correct their mistakes in an open learning environment. In particular, it is additional value not
only the cultivation of mathematical reasoning, analytical and productive geometric reasoning, innovative, creative and critical thinking, but also the activation of students’ motivations through the correlation of the activity with the real world, originality, creativity in order to develop the possibility of choices, the possibility of decision making, taking initiatives. Educational curriculum is essential to include innovative teaching and learning interdisciplinary practices, such the linking of the geometry with digital cultural heritage that development the knowledge and understanding and ability to use geometric properties and theorems; encourage the development and use of visual, collective reasoning and evidence and develop the awareness of the historical and cultural heritage of geometry in the society, and of the modern applications of geometry.

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